Mechanism Design and Market Institutions
Essay on School Matching

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Matching is one of the basic functions of the market. It decides the allocation of the resources among people. Among all matching problems, one class is rather interesting, namely school matching. The way of matching decides who can be admitted to school, and which school a student will finally enter. Because school matching has profound and long-lasting influence on students, parents and the society, studying the properties of different matching mechanism and choosing the most suitable one are objectives of economists.

In this essay, I first describe the school matching problem, state the purpose of researches on this topic, and discuss some criteria on matching mechanism. Then I present several matching mechanisms and their theoretical properties. After that, I give some discussion on application and experimental results of these school matching mechanism. Finally, concluding remarks will be given.

1   Introduction

1.1   School matching problem

The school matching problem concerns two set of agents, schools (colleges) and students (applicants). It was initially introduced by Gale and Shapley (1962) as the following situation: A school (college) has n applicants and can only accepts at most q. The school, after evaluating the applicants, need to send out offers to some of the applicants. The problem is that not all applicants who receive offer would reply to it. Therefore, the school needs to send more than q offers to students in order to receive approximately q acceptance.

This procedure generate too much guesswork because the school do not know whether the students apply other schools, what are the preferences of the students, and what are the

preferences of other schools. Therefore, the actual admission number can only be around desired q.

Difficulty are not only for colleges, but for students as well. They need to decide whether to respond to the current offer, or to wait for a preferred one. Since there are possibility of being declined by the preferred school, an applicant may make a sup-optimal decision.

The purpose of studying schooling matching problem is to overcome difficulties for both schools and students. A matching mechanism is aimed to provide an match between students and schools, i.e., who can go to school and which school she goes based on, ideally, true preferences of both parties. It should eliminate or minimize guesswork of both side, such that both side will achieve satisfactory result. It should also take into consideration the social efficiency and fairness.

1.2 Model description

Here I first give a formal model of school matching problem. There are two groups of individuals, m colleges (school) and n students (applicants), denoted by $C = \{c_1, c_2 \ldots c_m\}$ and $S = \{s_1, s_2 \ldots, s_n\}$ respectively. School i has a quota q_i, i.e., at most q_i students can be accepted. It is a two-sided matching, and, therefore, the agents of two groups do not overlap. Assume that the number of students is abundant enough for colleges to meet their quotas, i.e., $\sum_{i=1}^{m} q_i \leq n$.

The individuals in each side has a complete and transitive preferences over the individuals of the other sides. That is to say that, each college rank the students, while each student also rank the colleges. For example, the preference of college c_1 can be denoted as $P(c_1) = s_1, s_2, \ldots, s_k, c_1$. Here c_1 means that the college 1 would rather waste its quota than accept students ranked after itself. These unqualified students are omitted from the preference list of college c_1. Initially, we assume that the preferences are private information.[1]

For simplicity, I also assume that the preference are strict. If there is indifference in preference, the ties can be broken arbitrarily. For example, if a college is indifferent between two students, he can randomly rank these two students.

Moreover, if we set the quota q equal to 1 for each college, the students and the colleges are actually symmetric. It is a special case of schooling matching called "marriage model", and it is also the starting point of the analysis of Gale and Shapley (1962).

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[1]However, it is worthy pointing out that the preferences (priority) of public schools might be determined by announced rules, and thus preferences of schools can be regarded as public knowledge. This setting can affect the properties of mechanisms.

1.3 Two examples

Here I provide two simple examples of schooling matching problem. They will help us in later chapters understand theoretical properties of different matching mechanisms.

Example 1.

There are two colleges $C = \{c_1, c_2\}$ and three students $S = \{s_1, s_2, s_3\}$. Each school only admits 1 student, $q = 1$.

The Preferences of colleges are

$$P(c_1) = s_1, s_2, s_3$$
$$P(c_2) = s_3, s_1, c_2$$

The preferences of students are

$$P(s_1) = c_2, c_1$$
$$P(s_2) = c_1, s_2$$
$$P(s_3) = c_1, c_2$$

Example 2 (Gale and Shapley (1962))**.**

There are three colleges $C = \{c_1, c_2, c_3\}$ and three students $S = \{s_1, s_2, s_3\}$. Each school only admits 1 student, $q = 1$.

The Preferences of colleges are

$$P(c_1) = s_1, s_2, s_3$$
$$P(c_2) = s_2, s_3, s_1$$
$$P(c_3) = s_3, s_1, s_2$$

The preferences of students are

$$P(s_1) = c_2, c_3, c_1$$
$$P(s_2) = c_3, c_1, c_2$$
$$P(s_3) = c_1, c_2, c_3$$

1.4 Criteria

There are many mechanisms for matching schools and students. Each of them have certain properties and can fulfill certain purposes. Some of properties might be desirable. Therefore, we need to establish several criteria for matching procedures. Roth (1982) pointed out that any matching procedure depending on preference can be seen as two parts. One part is a mechanism to reveal the preferences of all agents, and the other part is to determine the outcome depending on the preferences revealed. If preferences are not truly revealed, resulting outcomes from stated preferences cannot process certain desirable properties with respect to true preferences. Therefore, the first criteria concerns incentive compatibility.

Criterion 1 (Strategy-proofness, incentive compatibility). *An assignment is strategy-proof if all agents prefer to reveal their true preferences. If only one side of the agents prefer to tell the truth, it is one-sided strategy-proof.*

Another criterion is the stability of matching. It eliminate the possibility that a pair of agents form a coalition and deviate the matching outcome.

Criterion 2 (Stability (Gale and Shapley, 1962)). *An assignment is unstable if there are two students s_1 and s_2 who are assigned to colleges c_1 and c_2. However, s_1 prefers c_2 to c_1 and c_2 prefers s_1 to s_2. Then, a transfer will improve the total benefits.*

Stability can also be regarded as fairness, namely no justified envy (Kojima and Manea, 2010). It means that there is no case in which student s_1 is matched to c_1 but prefer c_2, but c_2 accepts s_2 who ranks c_2 lower than s_1 does. When matching outcome is not compelled among all agents, stability is also a desirable criterion to prevent possible coalition.

The next criterion comes from social perspective. That is, whether the matching outcome is Pareto efficient.

Criterion 3 (Pareto efficiency (Roth, 1985)). *Some agents can improve their benefits while keeping others at least as good as current status.*

2 Mechanisms and theoretical properties

2.1 Deferred acceptance mechanism

Although college matching is a very important and common function of educational market, the economic theories of schooling matching was not systematically established by economists

until 1962 when Shapley and Gale published their leading paper College Admissions and the Stability of Marriage. In this paper, Gale and Shapley (1962) proposed the deferred acceptance mechanism for marriage matching and school matching. It requires all agents to state their preferences.

The analysis starts from the special case when $q = 1$. The algorithm has following steps.

Step 1: Individuals of one side, say students, make proposals to his favorite college. Each college who receives more than one proposals rejects all but its favorite students among all those who proposes to it. The school hold its favorite student on a string without immediately accept her.

Step 2: The students who were rejected in the step 1 can now propose to their second favorite school. After receiving the new proposals, the school chooses the favorite student from those who proposes in step 2 and the student on the string from step 1. This student will be put in the list, while other students are rejected.

In general at

Step k: The students who were rejected in step k-1 proposed to the last college in the preference lists. There are acceptance and rejection. Since a student cannot propose to the same college twice, the algorithm stops. The schools provides offer to the students on the string.

To see how this algorithm works, let us refer to **Example 1**. I use school proposing deferred acceptance algorithm here to find a match. In the first step, s_1 proposes to c_2, s_2 proposes to c_1, and s_3 proposes to c_1. c_1 holds s_2 on the string and rejects s_3, and c_2 holds s_1 on the string.

In the second step, s_3 proposes to c_2. c_2 holds s_3 on the string and rejects s_1.

In the third step, s_1 proposes to c_1, and c_1 hold s_1 on the string while it rejects s_2.

The algorithm terminates, and the schools accept the students on the string. The final match is $\mu = \{(c_1, s_1), (c_2, s_3), (\emptyset, s_2)\}$.

Now let us move on to see whether DA algorithms features certain criteria.

Theorem 1 (Gale and Shapley (1962)). *There always exists a stable set of matching. Resulting outcome produced by adopting deferred acceptance algorithm is stable.*

The proof of the theorem is simple. If c_1 and s_1 are not matched, but s_1 prefer c_1 to her own school. Then, s_1 must have proposed to c_1 at some steps before and was rejected by c_1 because c_1 prefers another student. In this way, we say the c_1 must prefer its matched student more than s_1. Thus, the deferred acceptance mechanism can eliminate instability.

Notably, (Gale and Shapley, 1962) also showed that the stable matching generated by deferred acceptance algorithm is one-sided optimal.

Criterion 4 (Optimal (Gale and Shapley, 1962))**.** *An assignment is optimal if every applicant is at least as well off under any other stable assignment.*

Theorem 2 (Gale and Shapley (1962), Roth (2008a))**.** *The matching produced by the deferred acceptance algorithm with one side proposing is the optimal stable matching for this side, i.e., every individual from proposing side likes this matching at least as well as other stable matching.*

This theorem indicates that the deferred acceptance algorithm favors the group of people who make a proposal. Let us see **Example 2**. There are three stable match:

The first matching outcome results from students proposing: $\mu_1 = \{(c_1, s_3), (c_2, s_1), (c_3, s_2)\}$.

The second matching outcome results from colleges proposing: $\mu_2 = \{(c_1, s_1), (c_2, s_2), (c_3, s_3)\}$.

The third matching outcome is also stable: $\mu_3\{(c_1, s_2), (c_2, s_3), (c_3, s_1)\}$.

We can see that the proposers are at least as well off under any other stable assignment using deferred acceptance algorithm. μ_1 is student-optimal stable matching, and μ_2 is school-optimal stable matching.

Gale and Shapley then extend 1-to-1 matching problem to many-to-1 matching problem, which is straightforward. Assume now the quotas of colleges, q, can be more than 1. We consider student proposing. The steps are as follows:

Step 1: Each students proposes to his favorite college. College i keeps the first q_i applicants on the waiting (if there are less than q_i students proposing to it, keep them all), and then rejects the others.

Step 2: The students who were rejected in the step 1 can now propose to their second favorite school. After receiving the new proposals, the college c_i chooses the favorite q_i student from those who proposes in step 2 and the student on the waiting list from step 1. These student will be on the waiting list, while other students are rejected.

In general at

Step k: The students who were rejected in step $k - 1$ proposed to the last college in the preference lists. There are acceptance and rejection. Since a student cannot propose to the same college twice, the algorithm stops. The schools provides offer to the students on the waiting list.

Later literature continued to investigate other properties of deferred acceptance mechanism. First of all, they want to see whether the DA algorithm is incentive compatible.

Theorem 3 ((Roth, 1982))**.** *No stable matching procedure for the general matching problem exists for which truthful revelation of preference is a dominant strategy for all agents.*

This theorem indicates that no matching procedure can generate both stable outcome and incentive compatible outcome. It directly means that deferred acceptance algorithm is not strategy-proof. See **Example 2** and consider students proposing. If, say, college 1 truncates its preference by reporting $P'(c_1) = s_1$. After 7 steps of iteration, the student-optimal stable matching under reported preferences is $\mu'_S = \{(c_1, s_1), (c_2, s_2), (c_3, s_3)\}$. The colleges are strictly better off by reporting fake preferences.

However, literature showed that truth-telling can still be dominant strategies for one side of the agents.

Theorem 4 (Dubins and Freedman (1981), Roth (1982), Roth (1985)). *It is a dominant strategy for each student to state his true preference when deferred acceptance algorithm is used and when students make proposers. In another words, the student-optimal stable matching is strategy-proof.*

This theorem sheds some light in application when preferences of schools are predetermined by announced rules. In this case, preferences of schools are actually publicly known. They will not behave strategically. The student-optimal stable matching is thus strategy-proof for all agents.

Literature also examined the another criterion, namely Pareto efficiency.

Criterion 5 (Weakly Pareto efficiency(Roth, 2008a)). *A matching μ is weakly pareto efficient for all students if there is no matching, including unstable ones, that all students strictly prefer it to μ.*

Theorem 5 (Roth (1982)). *The student-optimal matching is weakly Pareto optimal for all students. However, it is not strongly Pareto optimal for all students.*

Return to **Example 1**. The stable matching $\mu = \{(c_1, s_1), (c_2, s_3), (\emptyset, s_2)\}$ is dominated by an unstable matching $\mu' = \{(c_1, s_3), (c_2, s_1), (\emptyset, s_2)\}$ from the perspective of student side even though μ' is not stable, since s_1 and s_3 achieved their first choices while s_2 was not worse off.

2.2 Boston mechanism

Forty years after the deferred acceptance mechanism is proposed by Gale and Shapley, Abdulkadiroglu and Sönmez (2003) in their AER paper discusses another mechanism, Boston mechanism. In this mechanism, students are required to submit their private preference to educational center. The priorities of schools are determined by announced hierarchies, and

thus their quotas and preferences can be regarded as public information. The mechanism goes as follows[2].

Step 1: Each student proposes to the school she lists as her first choice. Each school accepts its applicants according to its preference until the quota is completely used. The remaining students are rejected.

Step 2: Those who are rejected in the step 1 proposes to the school they list as their second choice. Each school accepts its applicants according to its preference until the remaining quota is fully used.

In general at Step k: Those who are rejected in the step $k-1$ proposes to the school they list as their (k-1)th choice. Each school accepts its applicants according to its preference until the remaining quota is fully used.

The mechanism terminates either when there is no quota any more, or no students are rejected in this step.

The difference between the deferred acceptance mechanism and Boston mechanism lies in that the latter one does not never put any students on waiting lists, i.e., proposers in this step will not be compared to students in the waiting lists of last step.

See **Example 1**. There is only one step before matching is done if Boston mechanism is adopted. In this step, s_1 proposes to c_2, s_2 and s_3 proposes to c_1. Only s_3 is rejected. The resulting matching is $\mu = \{(c_1, s_2), (c_2, s_1), (\emptyset, s_3)\}$

This mechanism has following properties.

Theorem 6 (Abdulkadiroglu and Sönmez (2003)). *Boston mechanism is not strategy-proof.*

It is easy to see from **Example 1** that even if student s_3 ranked first in school c_2, she might still lose priority if he does not list school c_2 as her first choice. Thus, students and their parents might fake their true preferences, and improve the ranking of schools that give them high priority.

Moreover, Boston mechanism, theoretically, is neither stable nor Pareto efficient.

Theorem 7 (Abdulkadiroglu and Sönmez (2003)). *Boston mechanism is not stable.*

Theorem 8 (Abdulkadiroglu and Sönmez (2003)). *Boston mechanism is Pareto efficient if students present their true preferences. It is not Pareto efficient if students misrepresent their preferences.*

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[2]I rephrased the mechanism to make it more comparable to deferred acceptance mechanism.

2.3 Top trading cycles mechanism

In 1974, Shapley and Scarf (1974) introduced "top trading cycle" to find the competitive prices in the market. This concept was extended to school matching context by Abdulkadiroglu and Sönmez (2003). In this paper, the authors mentioned another matching procedure, namely top trading cycles mechanism. Given preferences of both sides, students are matched to schools with the following algorithm.

Step 1: Each student points to its announced favorite school, and each school points to its announced favorite student. This will generate at least a cycle. For example, $(s_1, c_1, s_2, c_3, \ldots, c_k)$, in which student s_1 points to college c_1, c_1 points to student s_2, and so on, until c_k points to s_1. Every student in the cycle is assigned to the school she points to.

In general at Step k: The rest students and colleges with quota continue to form cycles. Every student in the cycle is assigned to the school she points to.

The algorithm stops when no cycle can be formed.

Considering **Example 1**, in step 1, s_1 points to c_2, c_2 points to s_3, s_3 points to c_1, and c_1 points to s_1. There is a cycle and then s_1 is assigned to c_2 and s_3 is assigned to c_1. The resulting matching is $\mu = \{(c_1, s_3), (c_2, s_1), (\emptyset, s_2)\}$.

The top trading cycles mechanism has some different properties from deferred acceptance mechanism. These properties can also be seen from the resulting matching for **Example 1**.

First of all, other than DA algorithm, top trading cycles mechanism is Pareto efficient.

Theorem 9 (Abdulkadiroglu and Sönmez (2003)). *Top trading cycles mechanism is Pareto efficient with preferences of schools predetermined.*

Second, like DA algorithm, top trading cycles mechanism is strategy-proof.

Theorem 10 (Abdulkadiroglu and Sönmez (2003)). *Top trading cycles mechanism is strategy-proof with preferences of schools predetermined.*

However, top trading cycles mechanism produces unstable result.

Theorem 11 (Abdulkadiroglu and Sönmez (2003)). *Top trading cycles mechanism is unstable.*

3 Application and Experimental results

3.1 Application

3.1.1 Deferred acceptance mechanism for clearinghouses

Although deferred acceptance mechanism is introduced in 1962, similar ideas were adopted well before it. Roth (2008b) discussed the first application of the thought of deferred acceptance (DA) algorithm in doctor market[3].

The first jobs of newly-graduated doctors have great influence on their future careers. The jobs are also a great part of labor forces of hospital. In order to compete for graduates, hospitals tried to hire doctor students as early as possible. This resulted in the phenomenon that students were hired almost two years before they would graduate from medical school.

In order to cure this market failure, in 1945, American medical school agreed to not release students' information until certain date. This caused another problem that hospitals requested their candidates to reply the offers immediately, before the students receive about other offers. The shortened response time also caused chaos in the market.

In 1951, a centralized clearinghouse was adopted to coordinate the market, which is called National Resident Matching Program (NRMP) now. It requires both hospitals and graduates to submit their preferences, and uses an algorithm to produce a matching. Roth (1984) showed that this matching mechanism can result a hospital-optimal stable matching in the sense introduced by Gale and Shapley (1962).

Although the DA algorithm solved market failure in hospital-doctor matching, but, however, there are new challenges to this mechanism. One of them is that a growing number of married couples wish to be assigned to the same hospital. Roth (1984) showed that stable matching cannot be made in this situation. This caused high-qualified couples reluctant to participate the clearinghouse. Another issue is that the NRMP results in hospital-optimal stable matching, which pays little attention to the interest of the graduates.

Therefore, Roth was invited in 1995 to redesign the matching mechanism, and in 1999, Roth and Peranson (1999a) provided a new algorithm to always produce stable matching when there are couples. The new algorithm was consequently used by other labor market clearinghouses.

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[3]I include this example here because hospital matching problem is quite similar to a school matching problem to some extents.

3.1.2 Boston public school matching

The Boston mechanism was initially used by educational administration to allocate students to public schools. The problem of this system, as stated above, is that it is not safe for students and parents to state their true preferences. They might misrepresent their preferences and improve the ranking of schools which also give them higher priority. As recommended in the 2004-2005 BPS School Guide, "for a better choice of your 'first choice' school, consider choosing less popular schools."

In 2005, Abdulkadiroğlu et al. (2005) proposed two alternative mechanisms. One is deferred acceptance algorithm, and another one is top trading cycles mechanism. Because the schools' priority over students are not decided by schools but by the educational administration, there is no strategic issues for schools. Meanwhile, DA algorithm with student-proposing also ensure truth-telling. Therefore, both mechanism are strategy-proof. The choice between DA algorithm and TTC mechanism is just a trade-off between stability (fairness) and Pareto-efficiency on the student side.

Under the recommendation of Roth and his colleagues, the Boston school committee finally adopted the deferred acceptance algorithm with student proposing (Roth, 2008b).

3.1.3 New York school matching

The old system for school matching in New York is decentralized. Each student submits a preference list of 5 schools. The lists are sent to schools. The schools decide independently who they will accept and send out offer letters. Students reply to the offers after receiving mails. Then, school with empty seats send out offers for the second round. This procedure lasts for three round. Those who are not accepted by any school will be assigned to their zoned schools (Roth, 2008b).

This system generated a problem which is quite similar to what I described in introduction. There is too much guesswork and uncertainty for both schools and students. About 1,700 students received multiple offers and about 3,000 students received nothing.

Schooling matching problem in New York is different from that in Boston. It is because in New York, preferences of schools are not determined by authorities. Preferences over students are private information for schools, and schools can behave strategically. The analysis is more complicated than the Boston case.

Naturally, student proposing DA algorithm becomes a desirable mechanism for several reasons. First, student proposing DA algorithm provides student-optimal stable matching. It is envy-free and it cares more about the interest of students. Second, student proposing

DA algorithm is strategy-proof for students. Third, Roth and Peranson (1999b) and Kojima and Pathak (2009) showed that when the market is large, the proportion of colleges that misrepresent their preferences in student proposing DA algorithm converges to zero. It means the algorithm is almost strategy-proof for schools.

Roth (2008b) mentioned that after the New York City adopted this new school matching mechanism, the problem was solved. Only about 3,000 students did not receive offers. The submitted preferences also became more truthful.

4   Experimental results

In this chapter I will present two laboratory experiments that test the theoretical predictions of matching mechanisms.

The experiment carried out by Chen and Sönmez (2006) compares DA algorithm, TTC mechanism and Boston mechanism with criteria mentioned in introduction.

First, concerning strategy proof, Boston mechanism results in the least truth-telling, compared to other two mechanism. Top trading cycles mechanism results in weakly less truth-telling than DA algorithm. The authors showed that 70.8% of the participants got their reported first choice but only 28.5% of them got their true top choices. This indicates that applicants tends to improving the ranking of schools that give them high priority under Boston mechanism.

Second, concerning efficiency, DA algorithm always performs better than top trading cycles mechanism. Boston mechanism performs worse than TTC mechanism in the designed environment, but performs as well as DA algorithm in the random environment[4].

The main results of the experiment suggest that DA algorithm is better than the others.

Pais and Pintér (2008) also carried out experiments to compare these three mechanisms when participants have different level of information. Different from what was found by Chen and Sönmez (2006), they showed that TTC mechanism outperform DA algorithm in both truth-telling and efficiency, while DA algorithm does not show too much superiority over TTC mechanism in stability.

Their main results indicate, on the contrary, that TTC mechanism is superior over the other two.

These two experiments confirmed the theoretical predictions that Boston mechanism is the

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[4]In the designed environment, schools are assigned different characteristics to make them more realistic, and students' preferences over schools are adjusted with these characteristics. On the contrary, in the random environment, the payoff of attending each schools are only random integers

worst mechanism among the three in term of truth-telling, efficiency and stability. However, the experiments do not reach a agreement whether DA algorithm or TTC mechanism is better. Moreover, although theories predict that TTC mechanism outperforms DA algorithm in efficiency, the experimental results of Chen and Sönmez (2006) do not show this superiority.

5   Concluding remarks

School matching is an important issue. The matching may have profound influence on the future of the students, their family and society. Therefore, the study of different matching mechanism, as well as improving them towards desirable criteria, is what economists should do.

Current literature mainly focus on three criteria of matching mechanism. First, it should give students incentive to state their true preference. Second, it should be efficient on the student side. Third, it should eliminate or minimize unfairness. Given these criteria, two mechanisms are superior over others, namely student proposing deferred acceptance algorithm and top trading cycles mechanism.

Although theoretically DA algorithm results in more stable matching than TTC mechanism and the latter results in more efficient matching, current empirical results does not reach an agreement which one is better. Therefore, both DA algorithm and TTC mechanism be regarded as two best alternatives in school matching.

One more lesson we should also learn from the empirical application of these mechanisms is that there is never a "best" school matching mechanism. As time passes by, new problem rises and new criteria need to be taken into consideration. For example, DA algorithm successfully solved the market chaos for hospital-doctor matching in 1950th, but, however, it was challenged by growing number of married doctors. Therefore, new matching mechanisms need to be continuously explored to fulfill new social goals.

Future researchers can contribute to existing literature on school matching in following ways.

First, more matching mechanism currently in use can be studied. This should not be restricted in the US. What are the advantages and disadvantages of these mechanisms? What are the potential improvements? It is also important to take in to account the social and cultural background when doing analysis.

Consequently, second, other criteria for school matching mechanisms can be explored. These criteria may reflect the different social goals from era to era, and from culture to culture.

Third, the axiomatization of school matching problem can be considered. Do certain criteria uniquely determine the matching mechanism? It will be easy for educational authorities to select most the suitable matching mechanism if this axiomatization is done.

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